

CHAPTER

2

Term-I

INVERSE
TRIGONOMETRIC
FUNCTIONS

Syllabus

➤ Definition, range, domain, principal value branch.



STAND ALONE MCQs

(1 Mark each)

Q. 1. The value of $\sin^{-1}\left(\cos\frac{3\pi}{5}\right)$ is

(A) $\frac{\pi}{10}$

(B) $\frac{3\pi}{5}$

(C) $-\frac{\pi}{10}$

(D) $-\frac{3\pi}{5}$

[CBSE OD Set-I 2020]

Ans. Option (C) is correct.

Explanation:

$$\begin{aligned}
 &= \sin^{-1}\left[\cos\left(\frac{3\pi}{5}\right)\right] \\
 &= \sin^{-1}\left[\cos\left(\frac{\pi}{2} + \frac{\pi}{10}\right)\right] \\
 &= \sin^{-1}\left(-\sin\frac{\pi}{10}\right) \quad \left[\because \cos\left(\frac{\pi}{2} + x\right) = -\sin x\right] \\
 &= -\sin^{-1}\left(\sin\frac{\pi}{10}\right) \quad \left[\because \sin^{-1}(-x) = -\sin^{-1}x\right] \\
 &= -\frac{\pi}{10} \quad \left[\because \sin^{-1}(\sin x) = x, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right]
 \end{aligned}$$

Q. 2. The value of $\tan\left[\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right]$ is

(A) $\frac{3 + \sqrt{5}}{2}$

(B) $\frac{3 - \sqrt{5}}{2}$

(C) $\frac{-3 + \sqrt{5}}{2}$

(D) $\frac{-3 - \sqrt{5}}{2}$

Ans. Option (B) is correct.

Explanation:

$$x = \tan\left[\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right]$$

$$\text{Let } \cos^{-1}\frac{\sqrt{5}}{3} = \theta$$

$$\cos \theta = \frac{\sqrt{5}}{3}$$

$$\Rightarrow x = \tan \frac{1}{2} \theta$$

$$\Rightarrow x = \frac{\sin \frac{\theta}{2}}{\cos \frac{\theta}{2}}$$

$$\therefore \sin \frac{\theta}{2} = \frac{\sqrt{1 - \frac{\sqrt{5}}{3}}}{\sqrt{2}}$$

$$\Rightarrow \cos \frac{\theta}{2} = \frac{\sqrt{1 + \frac{\sqrt{5}}{3}}}{\sqrt{2}}$$

$$x = \frac{\sqrt{1 - \frac{\sqrt{5}}{3}}}{\sqrt{1 + \frac{\sqrt{5}}{3}}}$$

$$\begin{aligned}
 &= \frac{\sqrt{3-\sqrt{5}}}{\sqrt{3+\sqrt{5}}} \\
 &= \frac{\sqrt{3-\sqrt{5}}}{\sqrt{3+\sqrt{5}}} \times \frac{\sqrt{3-\sqrt{5}}}{\sqrt{3-\sqrt{5}}} \\
 &= \frac{3-\sqrt{5}}{\sqrt{(3)^2 - (\sqrt{5})^2}} \\
 &= \frac{3-\sqrt{5}}{\sqrt{9-5}} \\
 &= \frac{3-\sqrt{5}}{2}
 \end{aligned}$$

Q. 3. Which of the following is the principal value branch of $\cos^{-1}x$?

- (A) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (B) $\left[0, \frac{\pi}{2}\right]$
 (C) $[0, \pi]$ (D) $(0, \pi) - \left\{\frac{\pi}{2}\right\}$

Ans. Option (C) is correct.

Explanation: As we know that the principal value of $\cos^{-1}x$ is $[0, \pi]$.
 $y = \cos^{-1}x$

Q. 4. Which of the following is the principal value branch of $\operatorname{cosec}^{-1}x$?

- (A) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (B) $[0, \pi] - \left\{\frac{\pi}{2}\right\}$
 (C) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (D) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

Ans. Option (D) is correct.

Explanation : As we know that the principal value of $\operatorname{cosec}^{-1}x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$.
 $y = \operatorname{cosec}^{-1}x$

Q. 5. The value of $\sin^{-1}\left[\cos\left(\frac{33\pi}{5}\right)\right]$ is

- (A) $\frac{3\pi}{5}$ (B) $\frac{-7\pi}{5}$
 (C) $\frac{\pi}{10}$ (D) $\frac{-\pi}{10}$

Ans. Option (D) is correct.

Explanation: Let,
 $\sin^{-1}\left[\cos\left(\frac{33\pi}{5}\right)\right] = \sin^{-1}\left[\cos\left(6\pi + \frac{3\pi}{5}\right)\right]$
 $= \sin^{-1}\left[\cos\left(\frac{3\pi}{5}\right)\right]$
 $[\because \cos(2n\pi + \theta) = \cos\theta]$

$$\begin{aligned}
 &= \sin^{-1}\left[\cos\left(\frac{\pi}{2} + \frac{\pi}{10}\right)\right] \\
 &= \sin^{-1}\left(-\sin\frac{\pi}{10}\right) \\
 &\quad \left[\because \cos\left(\frac{\pi}{2} + x\right) = -\sin x\right] \\
 &= -\sin^{-1}\left(\sin\frac{\pi}{10}\right) \\
 &\quad [\because \sin^{-1}(-x) = -\sin^{-1}x] \\
 &= -\frac{\pi}{10} \\
 &\quad \left[\because \sin^{-1}(\sin x) = x, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right]
 \end{aligned}$$

Q. 6. The domain of function $\cos^{-1}(2x-1)$ is

- (A) $[0, 1]$ (B) $[-1, 1]$
 (C) $(-1, 1)$ (D) $[0, \pi]$

Ans. Option (A) is correct.

Explanation:
 We have $\cos^{-1}(2x-1)$
 $\Rightarrow -1 \leq 2x-1 \leq 1$ $[\because x \in [-1, 1]]$
 $\Rightarrow 0 \leq 2x \leq 2$
 $\Rightarrow 0 \leq x \leq 1$
 $\Rightarrow x \in [0, 1]$

Q. 7. The value of $\cos^{-1}\left(\cos\frac{3\pi}{2}\right)$ is

- (A) $\frac{\pi}{2}$ (B) $\frac{3\pi}{2}$
 (C) $\frac{5\pi}{2}$ (D) $\frac{7\pi}{2}$

Ans. Option (A) is correct.

Explanation: We have,
 $\cos^{-1}\left(\cos\frac{3\pi}{2}\right) = \cos^{-1}\left[\cos\left(2\pi - \frac{\pi}{2}\right)\right]$
 $\quad \left[\because \cos\left(2\pi - \frac{\pi}{2}\right) = \cos\frac{\pi}{2}\right]$
 $= \cos^{-1}\cos\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$
 $[\because \cos^{-1}(\cos x) = x, x \in [0, \pi]]$

Q. 8. The value of expression $2\sec^{-1}2 + \sin^{-1}\left(\frac{1}{2}\right)$ is

- (A) $\frac{\pi}{6}$ (B) $\frac{5\pi}{6}$
 (C) $\frac{7\pi}{6}$ (D) 1

Ans. Option (B) is correct.

Explanation: We have,

$$\begin{aligned}
 & 2\sec^{-1} 2 + \sin^{-1}\left(\frac{1}{2}\right) \\
 &= 2\sec^{-1} \sec \frac{\pi}{3} + \sin^{-1} \sin \frac{\pi}{6} \\
 &= 2 \times \frac{\pi}{3} + \frac{\pi}{6} \\
 &\quad \left[\because \sec^{-1}(\sec x) = x \text{ and } \sin^{-1}(\sin x) = x \right] \\
 &= \frac{4\pi + \pi}{6} \\
 &= \frac{5\pi}{6}
 \end{aligned}$$

Q. 9. What is the value of $\sec^2(\tan^{-1} 2)$

- (A) 1 (B) 4
(C) 5 (D) 3

Ans. Option (C) is correct.

Explanation :

$$\begin{aligned}
 \sec^2(\tan^{-1} 2) &= \sec^2(\sec^{-1} \sqrt{1+2^2}) \\
 &= \sec^2(\sec^{-1} \sqrt{5}) \\
 &= (\sqrt{5})^2 \\
 &= 5
 \end{aligned}$$

Q. 10. The principal value of

$$\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right) + 4\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) \text{ is}$$

- (A) $\frac{\pi}{3}$ (B) $\frac{\pi}{6}$
(C) $\frac{4\pi}{3}$ (D) $\frac{3\pi}{4}$

Ans. Option (C) is correct.

Explanation :

$$\begin{aligned}
 & \cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right) + 4\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) \\
 &= \cos^{-1}\left(\cos \frac{\pi}{3}\right) + 2\sin^{-1}\left(\sin \frac{\pi}{6}\right) + 4\tan^{-1}\left(\tan \frac{\pi}{6}\right) \\
 &= \frac{\pi}{3} + 2 \times \frac{\pi}{6} + 4 \times \frac{\pi}{6} \\
 &= \frac{2\pi + 2\pi + 4\pi}{6} \\
 &= \frac{8\pi}{6} \\
 &= \frac{4\pi}{3}
 \end{aligned}$$

Q. 11. The principal value of $\cot^{-1}(-\sqrt{3})$ is

- (A) $\frac{5\pi}{6}$ (B) $\frac{\pi}{2}$

- (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{3}$

Ans. Option (A) is correct.

Explanation :

$$\begin{aligned}
 \text{Let } \cot^{-1}(-\sqrt{3}) &= \theta \\
 \Rightarrow \cot \theta &= -\sqrt{3} \\
 \Rightarrow \cot \theta &= -\cot \frac{\pi}{6} \\
 &= \cot\left(\pi - \frac{\pi}{6}\right) \\
 \Rightarrow \cot \theta &= \cot \frac{5\pi}{6} \\
 \Rightarrow \theta &= \frac{5\pi}{6} \in (0, \pi) \\
 \therefore \text{Principal value of } \cot^{-1}(-\sqrt{3}) &\text{ is } \frac{5\pi}{6}
 \end{aligned}$$

Q. 12. Domain of $\sin^{-1}x$ is:

- (A) $[-1, \infty)$ (B) $[-1, 1]$
(C) $(-1, 1)$ (D) None of these.

Ans. Option (B) is correct.

Explanation : Domain of $\sin^{-1}x$ is $[-1, 1]$

Q. 13. Range of $\cos^{-1}x$ is:

- (A) $\left[0, \frac{\pi}{2}\right]$ (B) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
(C) $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ (D) $[0, \pi]$

Ans. Option (D) is correct.

Explanation : The branch with range $[0, \pi]$ is called the principal value branch of the function $\cos^{-1}x$.

Q. 14. Domain of $\sec^{-1}x$ is:

- (A) $\mathbb{R} - (-1, 1)$ (B) \mathbb{R}
(C) $[-1, 1]$ (D) $\mathbb{R} - (0, 1)$

Ans. Option (A) is correct.

Q. 15. The value of $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$ is:

- (A) π (B) $-\frac{\pi}{3}$
(C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$

Ans. Option (B) is correct.

$$\begin{aligned}
 \text{Explanation : } \tan^{-1}(\sqrt{3}) - \sec^{-1}(-2) \\
 &= \frac{\pi}{3} - \frac{2\pi}{3} \\
 &= -\frac{\pi}{3}
 \end{aligned}$$



ASSERTION AND REASON BASED MCQs

(1 Mark each)

Directions : In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.

- (A) Both A and R are true and R is the correct explanation of A
- (B) Both A and R are true but R is NOT the correct explanation of A
- (C) A is true but R is false
- (D) A is false and R is True

Q. 1. Assertion (A): $\sin^{-1}\left(\sin \frac{2\pi}{3}\right) = \frac{2\pi}{3}$

Reason (R): $\sin^{-1}(\sin \theta) = \theta$, if $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Ans. Option (D) is correct.

Explanation:

The principal value branch of $\sin^{-1}x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Let $x = \sin \theta \Rightarrow \theta = \sin^{-1}x$

$\sin^{-1}(\sin \theta) = \sin^{-1}x = \theta$

$\sin^{-1}(\sin \theta) = \theta$, if $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Hence R is true.

$\sin^{-1}\left(\sin \frac{2\pi}{3}\right) \neq \frac{2\pi}{3}$, since $\frac{2\pi}{3} \notin \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Hence A is false.

Q. 2. Assertion (A): Range of $\tan^{-1}x$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Reason (R): Domain of $\tan^{-1}x$ is \mathbb{R} .

Ans. Option (B) is correct.

Explanation: Domain of $\tan x$ is the set $\{x : x \in \mathbb{R} \text{ and } x \neq (2n+1)\frac{\pi}{2}, n \in \mathbb{Z}\}$ and Range is \mathbb{R} .

$\Rightarrow \tan x$ is not defined for odd multiples of $\frac{\pi}{2}$.

If we restrict the domain of tangent function to $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, then it is one-one and onto with its range as \mathbb{R} . Actually $\tan x$ restricted to any of the intervals $\left(-\frac{3\pi}{2}, -\frac{\pi}{2}\right), \left(-\frac{\pi}{2}, \frac{\pi}{2}\right), \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ etc., is bijective and its range is \mathbb{R} .

Thus $\tan^{-1}x$ can be defined as a function whose domain is \mathbb{R} and range could be any of the intervals $\left(-\frac{3\pi}{2}, -\frac{\pi}{2}\right), \left(-\frac{\pi}{2}, \frac{\pi}{2}\right), \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ and soon.

\therefore Both A and R are true but R is not correct explanation of A.

Q. 3. Assertion (A): Principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ is $\frac{\pi}{4}$

Reason (R): Principal value of $\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)$ is $\frac{\pi}{3}$

Ans. Option (C) is correct.

Explanation:

$$\begin{aligned} \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) &= \sin^{-1}\left(\sin \frac{\pi}{4}\right) \\ &= \frac{\pi}{4} \end{aligned}$$

$$\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right) = y$$

$$\cot y = \frac{-1}{\sqrt{3}}$$

$$= -\cot\left(\frac{\pi}{3}\right)$$

$$= \cot\left(\pi - \frac{\pi}{3}\right)$$

$$= \cot\left(\frac{2\pi}{3}\right)$$

$$\Rightarrow \cot^{-1}\left(\frac{-1}{\sqrt{3}}\right) = \frac{2\pi}{3}$$

Hence Assertion is correct and Reason is incorrect.

Q. 4. Assertion (A): Range of $\cot^{-1}x$ is $(0, \pi)$

Reason (R): Domain of $\tan^{-1}x$ is \mathbb{R} .

Ans. Option (B) is correct.

Q. 5. Assertion (A): Principal value of $\cos^{-1}(1)$ is π

Reason (R): Value of $\cos 0^\circ$ is 1

Ans. Option (D) is correct.

Explanation: In case of Assertion:

$$\cos^{-1}(1) = y$$

$$\cos y = 1$$

$$\cos y = \cos 0^\circ \quad [\because \cos 0^\circ = 1]$$

$$\therefore y = 0$$

\Rightarrow Principal value of $\cos^{-1}(1)$ is 0

Hence Assertion is incorrect.

Reason is correct.

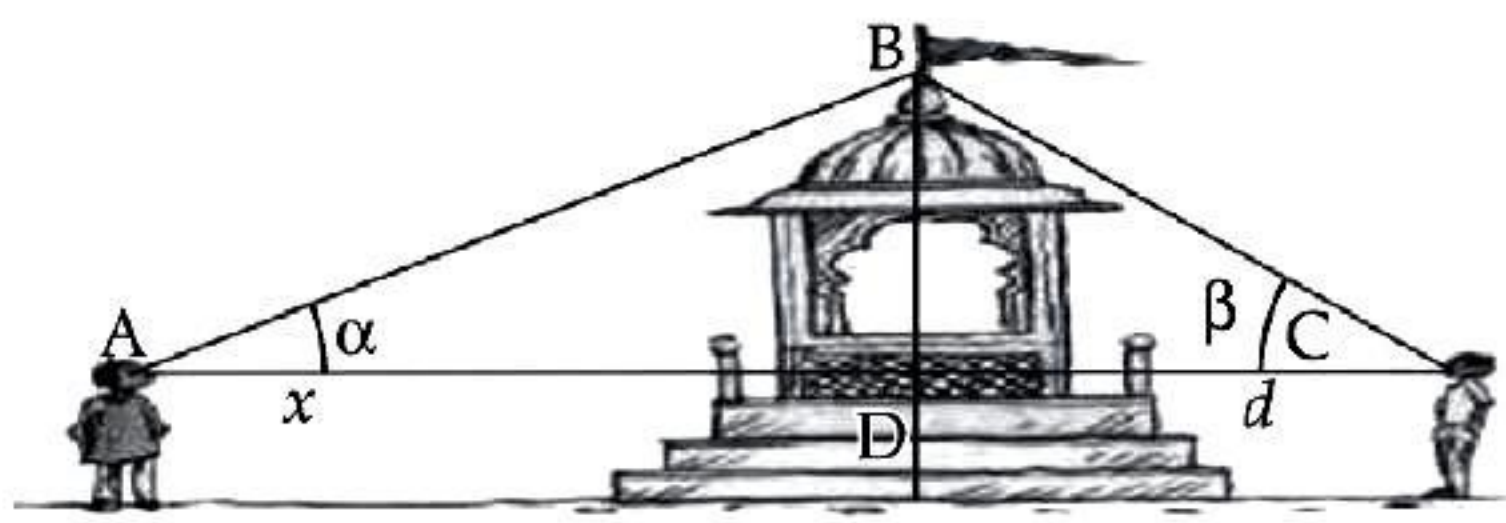




CASE-BASED MCQs

Attempt any four sub-parts from each question.
Each sub-part carries 1 mark.

I. Read the following text and answer the following questions on the basis of the same:



Two men on either side of a temple of 30 metres high observe its top at the angles of elevation α and β respectively. (as shown in the figure above). The distance between the two men is $40\sqrt{3}$ metres and the distance between the first person A and the temple is $30\sqrt{3}$ meters. [CBSE QB-2021]

Q. 1. $\angle CAB = \alpha =$

- (A) $\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (B) $\sin^{-1}\left(\frac{1}{2}\right)$
(C) $\sin^{-1}(2)$ (D) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

Ans. Option (B) is correct.

Explanation: In $\triangle BDA$

$$\sin \alpha = \frac{BD}{AB}$$

$$\begin{aligned} AB^2 &= AD^2 + BD^2 \\ &= (30\sqrt{3})^2 + (30)^2 \\ &= (60)^2 \end{aligned}$$

$$AB = 60\text{m}$$

Now, $\sin \alpha = \frac{30}{60}$

$$\sin \alpha = \frac{1}{2}$$

i.e. $\angle CAB = \alpha = \sin^{-1}\left(\frac{1}{2}\right)$

Q. 2. $\angle CAB = \alpha =$

- (A) $\cos^{-1}\left(\frac{1}{5}\right)$ (B) $\cos^{-1}\left(\frac{2}{5}\right)$
(C) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (D) $\cos^{-1}\left(\frac{4}{5}\right)$

Ans. Option (C) is correct.

Explanation: In $\triangle BDA$

$$\cos \alpha = \frac{AD}{AB}$$

$$\cos \alpha = \frac{30\sqrt{3}}{60}$$

$$\alpha = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\therefore \angle CAB = \alpha = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

Q. 3. $\angle BCA = \beta =$

- (A) $\tan^{-1}\left(\frac{1}{2}\right)$ (B) $\tan^{-1}(2)$
(C) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (D) $\tan^{-1}(\sqrt{3})$

Ans. Option (D) is correct.

Explanation:

$$\begin{aligned} DC &= AC - AD \\ &= 40\sqrt{3} - 30\sqrt{3} \\ &= 10\sqrt{3}\text{m} \end{aligned}$$

In $\triangle BDC$

$$\tan \beta = \frac{BD}{DC} = \frac{30}{10\sqrt{3}} = \sqrt{3}$$

$$\angle BCA = \beta = \tan^{-1}(\sqrt{3})$$

Q. 4. $\angle ABC =$

- (A) $\frac{\pi}{4}$ (B) $\frac{\pi}{6}$
(C) $\frac{\pi}{2}$ (D) $\frac{\pi}{3}$

Ans. Option (C) is correct.

Explanation: Since,

$$\sin \alpha = \frac{1}{2}$$

i.e.,

$$\sin \alpha = \sin 30^\circ$$

$$\left[\because \sin 30^\circ = \frac{1}{2} \right]$$

\therefore

$$\alpha = 30^\circ$$

we, have

$$\tan \beta = \sqrt{3}$$

$$\tan \beta = \tan 60^\circ$$

\therefore

$$\beta = 60^\circ$$

Now, In $\triangle ABC$

$$\angle ABC + \angle BCA + \angle CAB = 180^\circ$$

$$\angle ABC + 60^\circ + 30^\circ = 180^\circ$$

$$\angle ABC = 90^\circ$$

\therefore

$$\angle ABC = \frac{\pi}{2}$$



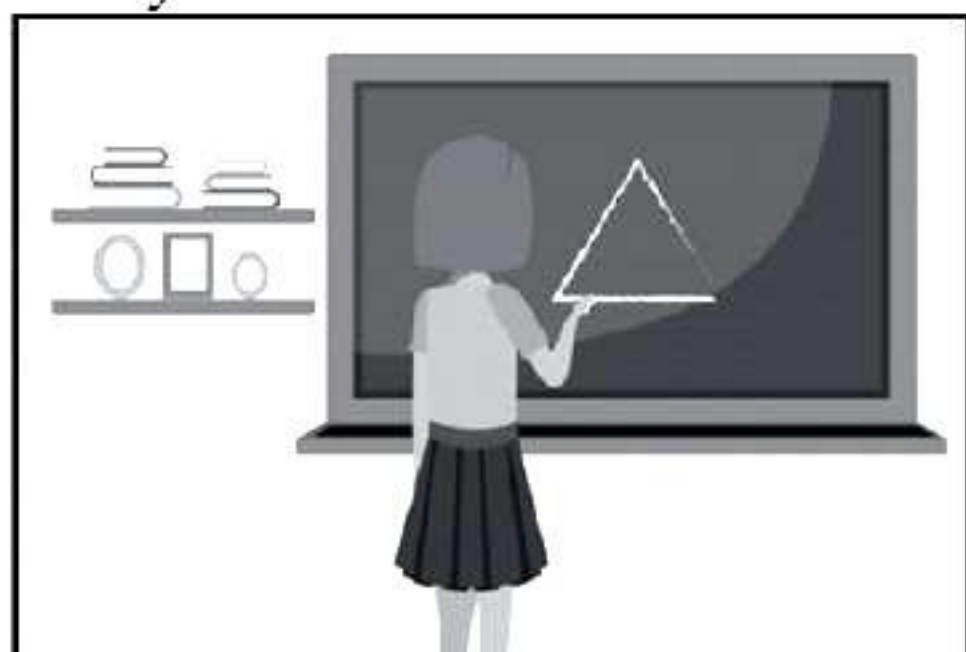
Q. 5. Domain and Range of $\cos^{-1} x =$

- (A) $(-1, 1), (0, \pi)$ (B) $[-1, 1], (0, \pi)$
 (C) $[-1, 1], [0, \pi]$ (D) $(-1, 1), \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Ans. Option (C) is correct.

II. Read the following text and answer the following questions on the basis of the same:

In the school project Sheetal was asked to construct a triangle and name it as ABC . Two angles A and B were given to be equal to $\tan^{-1} \frac{1}{2}$ and $\tan^{-1} \frac{1}{3}$ respectively.

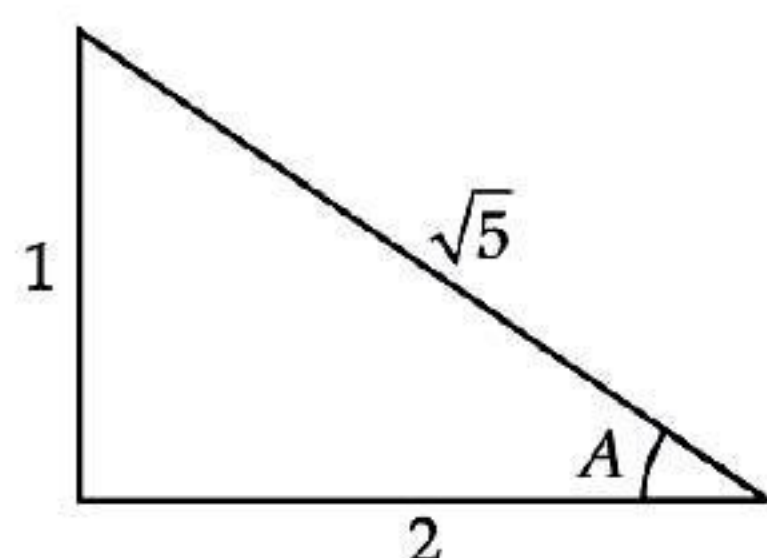


Q. 1. The value of $\sin A$ is _____.

- (A) $\frac{1}{2}$ (B) $\frac{1}{3}$
 (C) $\frac{1}{\sqrt{5}}$ (D) $\frac{2}{\sqrt{5}}$

Ans. Option (C) is correct.

Explanation:



$$A = \tan^{-1} \frac{1}{2}$$

$$\Rightarrow \tan A = \frac{1}{2}$$

$$\therefore \sin A = \frac{1}{\sqrt{5}}$$

Q. 2. $\cos(A + B + C) =$ _____.

- (A) 1 (B) 0
 (C) -1 (D) $\frac{1}{2}$

Ans. Option (C) is correct.

Explanation: Since ABC is a triangle,

$$\begin{aligned} \therefore A + B + C &= 180^\circ \\ \cos(A + B + C) &= \cos 180^\circ \\ &= -1 \end{aligned}$$

Q. 3. If $B = \cos^{-1} x$, then $x =$ _____.

- (A) $\frac{1}{\sqrt{5}}$ (B) $\frac{3}{\sqrt{10}}$

- (C) $\frac{1}{\sqrt{10}}$ (D) $\frac{2}{\sqrt{5}}$

Ans. Option (B) is correct.

Explanation:

$$\begin{aligned} \text{Given } B &= \tan^{-1} \frac{1}{3} \\ \Rightarrow \tan B &= \frac{1}{3} \\ \therefore \cos B &= \frac{3}{\sqrt{10}} \\ B &= \cos^{-1} \frac{3}{\sqrt{10}} \\ \Rightarrow x &= \frac{3}{\sqrt{10}} \end{aligned}$$

Q. 4. If $A = \sin^{-1} x$, then the value of x is:

- (A) $\frac{1}{\sqrt{5}}$ (B) $\frac{2}{\sqrt{5}}$
 (C) $\frac{1}{\sqrt{10}}$ (D) $\frac{3}{\sqrt{10}}$

Ans. Option (A) is correct.

Explanation:

$$\begin{aligned} A &= \tan^{-1} \frac{1}{2} \\ \Rightarrow \tan A &= \frac{1}{2} \\ \therefore \sin A &= \frac{1}{\sqrt{5}} \\ A &= \sin^{-1} \left(\frac{1}{\sqrt{5}} \right) \\ \Rightarrow x &= \frac{1}{\sqrt{5}} \end{aligned}$$

Q. 5. The third angle, $\angle C =$ _____.

- (A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$
 (C) $\frac{\pi}{3}$ (D) $\frac{3\pi}{4}$

Ans. Option (D) is correct.

Explanation:

$$\begin{aligned} \angle C &= \pi - (A + B) \\ &= \pi - \frac{\pi}{4} \\ &= \frac{3\pi}{4} \end{aligned}$$

III. Read the following text and answer the following questions on the basis of the same:

The value of an inverse trigonometric functions which lies in the range of Principal branch is called the principal value of that inverse trigonometric functions.

Q. 1. Principal value of $\sin^{-1}\left(\frac{1}{2}\right)$ is

- (A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$
(C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$

Ans. Option (A) is correct.

Explanation:

$$\begin{aligned}\sin^{-1}\left(\frac{1}{2}\right) &= y \\ \sin y &= \frac{1}{2}\end{aligned}$$

Principal value branch of \sin^{-1} is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$$\text{and } \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

$$\Rightarrow \text{Principal value of } \sin^{-1}\left(\frac{1}{2}\right) \text{ is } \frac{\pi}{6}$$

Q. 2. Principal value of $\tan^{-1}(1)$

- (A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$
(C) π (D) $\frac{\pi}{3}$

Ans. Option (A) is correct.

Explanation:

$$\begin{aligned}\tan^{-1}(1) &= \tan^{-1}\left(\tan\frac{\pi}{4}\right) \\ &= \frac{\pi}{4}\end{aligned}$$

Q. 1. Principal value of $\cot^{-1}(\sqrt{3})$ is :

- (A) $\frac{\pi}{3}$ (B) π

- (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{2}$

Ans. Option (C) is correct.

Explanation:

$$\begin{aligned}\cot^{-1}(\sqrt{3}) &= \cot^{-1}\left(\cot\frac{\pi}{6}\right) \\ &= \frac{\pi}{6}\end{aligned}$$

Q. 4. Principal value of $\sin^{-1}(1) + \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ is

- (A) 2π (B) π
(C) $\frac{3\pi}{4}$ (D) $\frac{\pi}{3}$

Ans. Option (C) is correct.

Explanation:

$$\begin{aligned}\sin^{-1}(1) + \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) &= \frac{\pi}{2} + \frac{\pi}{4} \\ &= \frac{3\pi}{4}\end{aligned}$$

Q. 5. Principal value of $2\cos^{-1}(1) + 5\tan^{-1}(1)$ is:

- (A) $\frac{3\pi}{4}$ (B) $\frac{\pi}{4}$
(C) $\frac{\pi}{2}$ (D) $\frac{5\pi}{4}$

Ans. Option (D) is correct.

Explanation:

$$\begin{aligned}2\cos^{-1}(1) + 5\tan^{-1}(1) &= 2 \times 0 + 5 \times \frac{\pi}{4} \\ &= \frac{5\pi}{4}\end{aligned}$$

□□